Kirchhoff’s law  (Item No.: P2410500)

Curricular Relevance


Difficulty | Preparation Time | Execution Time | Recommended Group Size
--- | --- | --- | ---
Difficult | 1 Hour | 2 Hours | 2 Students

Additional Requirements: Experiment Variations:

**Keywords:**
Kirchhoff’s laws, induction law, Maxwell equations, current, voltage, resistance, parallel connection, series connection, potentiometer

Overview

Short description

First Kirchhoff’s laws are verified by measuring current, voltage and resistance in series and parallel circuits. The Wheatstone bridge circuit is used to determine unknown resistances more precisely.

![Experimental set-up for determining unknown resistances](image)

**Fig. 1:** Experimental set-up for determining unknown resistances
Equipment

<table>
<thead>
<tr>
<th>Position</th>
<th>Material</th>
<th>Order nr.</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PHYWE Power supply 0-12 V DC/ 6 V, 12 V AC, 230 V</td>
<td>13505-93</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Digital multimeter 2005</td>
<td>07129-00</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Connection box</td>
<td>06030-23</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Resistor 220 Ohm, 1W, G1</td>
<td>39104-64</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>Resistor 330 Ohm, 1W, G1</td>
<td>39104-13</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>Resistor 2.2 kOhm, 1W, G1</td>
<td>39104-23</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>Resistor 3.3 kOhm, 1W, G1</td>
<td>39104-25</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>Resistor 100 Ohm, 1W, G1</td>
<td>39104-63</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>Resistor 470 Ohm, 1W, G1</td>
<td>39104-15</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>Resistor 1 kOhm, 1W, G1</td>
<td>39104-19</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
<td>Resistor 4.7 kOhm, 1W, G1</td>
<td>39104-27</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>Resistor 10 kOhm, 1W, G1</td>
<td>39104-30</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>Connecting plug white 19 mm pitch</td>
<td>39170-00</td>
<td>3</td>
</tr>
<tr>
<td>14</td>
<td>Connecting cord, 32 A, 250 mm, red</td>
<td>07360-01</td>
<td>2</td>
</tr>
<tr>
<td>15</td>
<td>Connecting cord, 32 A, 250 mm, blue</td>
<td>07360-04</td>
<td>2</td>
</tr>
</tbody>
</table>

Tasks

1. Task: Verify Kirchhoff’s laws by measuring current and voltage for series and parallel connected resistors for each resistor as well as the total values. From these measurements calculate the partial and total resistances.

2. Task: Determine unknown resistances by the use of the Wheatstone bridge circuit.

Set-up and procedure

Set-up

Task 1

The circuit is set up as shown in Fig. 2. The digital multimeter has to be used as Voltmeter and Ampèremeter alternatingly. Try different resistors in order to verify Kirchhoff’s laws.

Task 2

The resistors that have to be determined need to be disguised. You can use non-transparent tape to cover the resistors’ cases. Set up the experiment according to Figs. 1 and 3. The digital multimeter is connected in series with \( R_1 \) and \( R_2 \) in order to measure the current \( I_g \). Choose the lowest possible measuring range and set the measuring mode to direct current (DC). Be careful to plug the connection cords into the correct sockets of the multimeter.

Procedure

Task 1

Before switching on the power supply, make sure that both adjustments of current and voltage are tuned down to zero. After switching on the power supply first tune the current until the green LED goes out. Then carefully tune up the voltage to a maximum of 3 V. Measure the current in the unbranched part of the circuit. Replace \( c1 \) with the multimeter and measure the partial current \( I_1 \). Continue with the measurement of \( I_2 \) at \( c2 \). Measure the applied voltage directly at the source. Afterwards, insert a third resistor \( R_3 \) in series with \( R_1 \). Measure the partial voltages \( U_1 \) and \( U_3 \) across \( R_1 \) and \( R_3 \) respectively.

Note: To perform the measurements you will have to build the circuit for each measurement individually.

Task 2

Before switching on the power supply, make sure that both adjustments of current and voltage are tuned down to zero. After switching on the power supply first tune the current until the green LED goes out. Then carefully tune up the voltage to a maximum of 1 V. If the green LED lights up again you have to adjust the current. Keep an eye on the digital multimeter measuring the current in the circuit and keep the current well below 1 A. In order to determine the unknown resistance change \( R_3 \) until the current through G vanishes. You may try single resistors as well several resistors connected in series to vary \( R_3 \). Note down the combinations at which the current becomes zero or at least reaches a minimum.
Fig. 2: Schematic circuit for Task 1.

Fig. 3: Wheatstone bridge circuit with resistances $R_1$ and $R_2$ instead of Wheatstone bridge.

Fig. 4: Schematic circuit for the Wheatstone bridge. If actually using a Wheatstone bridge, $R_1$ and $R_2$ are variable with $R_1 + R_2 = \text{const.}$

Theory and evaluation

Theory

Task 1

With branched circuits, in the steady-state condition, Kirchhoff’s 1st law applies at every junction point:

$$\sum_i I_i = 0 \quad (1)$$
where $I_k$ are the currents leading to or from the junction point. This means, that in every junction point, the charge is conserved. It is customary to take $I_k$ as negative if the corresponding current in the $k$-th conductor is flowing away from the junction point.

For every closed loop $C$ in a network of linear conductors, in the steady-state condition, Kirchhoff’s 2nd law applies:

$$\sum_k U_k = 0 \quad (2)$$

where $U_k$ the voltage in the $k$-th conductor. This is a special case of the induction law as it applies only for constant magnetic flows. More precisely, it is a conclusion of the 1st and 3rd of Maxwell’s equations. It means that, in a closed loop, the electrical energy is conserved.

From these laws follow some conclusions how current, resistance and voltage behave in parallel and series circuits:

1. series connection of several conductors

   - $I_n = I_m$: Through every conductor flows the same current.
   - $U_{tot} = \sum_n U_n$: The voltages across the individual conductors sum up to the total voltage across the circuit.
   - $R_{tot} = \sum_n R_n$: The resistances across the individual conductors sum up to the total resistance across the circuit.

   From this follows:

   $$\frac{U_{tot}}{R_{tot}} = \frac{U_n}{R_n} \quad \text{as well as} \quad \frac{R_n}{R_{tot}} = \frac{U_n}{U_{tot}} \quad (3)$$

   where $I = \frac{U}{R}$ is Ohm’s law.

2. parallel connection of several conductors

   - $I = \sum_n I_n$: The currents of the individual conductors sum up to the total current through the circuit.
   - $U_n = U_m$: The voltage across any conductor is the same.
   - $\frac{1}{R_{tot}} = \sum_n \left(\frac{1}{R_n}\right)$: The resistances across the individual conductors sum up to the total resistance across the circuit.

   Task 2

   In principal an unknown resistance can be determined by measuring current and voltage across the resistance. The finite intrinsic resistances of the instruments would introduce significant errors. To avoid such errors the measurement has to be done current-free.

   In a Wheatstone bridge circuit the unknown resistance $R_x$ is connected to three resistances (see Fig. 4) of which at least one is variable.

   In this experiment $R_3$ is adjusted in such a manner that no current is flowing through the instrument $G$ (alignment of the bridge) which means the voltage across $G$ vanishes as well. In this case the voltages across $R_2$ and $R_1$ are equal as well as across $R_3$ and $R_2$. Also, as no current is flowing through $G$, the same current is flowing through $R_3$ and $R_x$ on the one hand (denoted as $I_1$) and through $R_1$ and $R_2$ on the other hand ($I_2$). This results into the following equations:

   $$I_1 R_x = I_2 R_1 \quad \text{and} \quad I_1 R_3 = I_2 R_2$$

   Division of these relations yields eq. (4) which computes the unknown resistance $R_x$. 
\[ R_z = R_3 \cdot \left( \frac{R_1}{R_2} \right) \quad (4) \]

The voltage at the source is unimportant and may even be time-dependent.

**Note:** In a customary Wheatstone bridge, \( R_1 \) and \( R_2 \) are two parts of a wire separated by a sliding contact (similar to a potentionmeter). So \( R_1 \) and \( R_2 \) are both variable where the sum is the total resistance of the wire. However, in this experiment \( R_1 \) and \( R_2 \) are constant and \( R_3 \) is changed.

## Evaluation

### Evaluation and results

In the following the evaluation of the obtained values is described with the help of example values. Your results may vary from those presented here.

**Task 1**

In order to verify Kirchhoff's laws compare the measured values with the theoretical values obtained from eqs. (1) to (3). To some degree, deviations from the computed results have to be expected, e.g. the connection cords have a non-zero resistance and therefore contribute to the total resistance of the circuit.

**Task 2**

Having found the appropriate resistances for \( R_3 \) that will minimize the current through \( G \) use eq. (4) to calculate the unknown resistances. Tab 1 shows an example of combinations of \( R_3 \) and \( R_x \) for the setup of this experiment.

**Note:**

Instead of single resistors you can use several potentionmeter with different ranges connected in series or a decade resistance box (06194-10) as the third resistance \( R_3 \).

### Tab. 1: Resistances determined via the Wheatstone bridge circuit with \( R_1 = 1k\Omega \) and \( R_2 = 10k\Omega \).

<table>
<thead>
<tr>
<th>( R_3 ) (k( \Omega ))</th>
<th>( R_x ) (k( \Omega ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1 1.1</td>
<td>0.1</td>
</tr>
<tr>
<td>X2 2.2</td>
<td>0.22</td>
</tr>
<tr>
<td>X3 3.3</td>
<td>0.33</td>
</tr>
<tr>
<td>X4 4.7</td>
<td>0.47</td>
</tr>
</tbody>
</table>