Charging curve of a capacitor and switch-on behaviour of an inductance with Cobra4 Xpert-Link

(Item No.: P2420264)

Curricular Relevance

<table>
<thead>
<tr>
<th>Area of Expertise:</th>
<th>Physics</th>
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<tr>
<td>Education Level:</td>
<td>University</td>
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<tr>
<td>Topic:</td>
<td>Electricity and Magnetism</td>
</tr>
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<td>Subtopic:</td>
<td>Capacitance, Dielectrics, Electric Energy, Storage</td>
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<td>Experiment:</td>
<td>Charging curve of a capacitor and switch-on behaviour of an inductance with Cobra4 Xpert-Link</td>
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Difficulty | Preparation Time | Execution Time | Recommended Group Size
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Difficult | 🌟🌟🌟🌟🌟 | 🌟🌟🌟🌟 | 🧑‍🔬_ONE Student

Additional Requirements:

Experiment Variations:

Keywords:
Charging, discharging, time constant, exponential function, half life

Introduction

Overview

1. A circuit with a constant voltage source, a switch, a resistor and a capacitor in series is switched on. The development of voltage on the resistor and current through the circuit after switching on is recorded over the time.

2. A circuit with a constant voltage source, a switch, a resistor and an inductance in series is switched on. The development of voltage on the inductance and current through the circuit after switching on is recorded over the time.

Fig. 1: Experimental setup. Here: Inductance measurement.
### Equipment

<table>
<thead>
<tr>
<th>Position No.</th>
<th>Material</th>
<th>Order No.</th>
<th>Quantity</th>
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<tbody>
<tr>
<td>1</td>
<td>Cobra4 Xpert-Link</td>
<td>12625-99</td>
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</tr>
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<td>2</td>
<td>Cobra4 Xpert-Link set of cables</td>
<td>12625-10</td>
<td>1</td>
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<td>3</td>
<td>Connection box</td>
<td>06030-23</td>
<td>1</td>
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<tr>
<td>4</td>
<td>Resistor 47 Ohm, 1W, G1</td>
<td>39104-62</td>
<td>1</td>
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<td>5</td>
<td>Resistor 1 kOhm, 1W, G1</td>
<td>39104-19</td>
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<tr>
<td>6</td>
<td>Electr. capacitor 47µF/35V</td>
<td>39105-24</td>
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<td>7</td>
<td>Electrol. capacitor 10µF/35V</td>
<td>39105-28</td>
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<td>8</td>
<td>Short-circuit plug, white</td>
<td>06027-06</td>
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<tr>
<td>9</td>
<td>Coil, 900 turns</td>
<td>06512-01</td>
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<td>10</td>
<td>Software measureLAB</td>
<td>14580-61</td>
<td>1</td>
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<td></td>
<td>PC, Windows® 7 or higher</td>
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### Tasks

1. Determine capacitance and resistance of the circuit from the measured curves.
2. Determine inductance and resistance of the circuit from the measured curves.
Set-up and procedure

1. Capacitance measurement

Set up your equipment according to Fig. 2.

![Fig. 2: Capacitance measurement circuit.](image)

In accordance to the Kirchhoff rules, the measurement input of the current is connected in series while the voltage drop at the capacitor is measured in a parallel connection.

In this circuit, the resistor is limiting the current values to the selected measurement range (here: 10 mA). Choosing a lower resistance value will result in a lesser limitation, thus requiring a different measurement range (100 mA or more), which will result in a loss of measurement precision. For this reason, a resistance value of 1 kΩ is chosen, optimizing the setup for a measurement range of 10 mA.

Connect the Xpert-Link to your computer and switch on the Xpert-Link.

Start the software “measureLAB”, and choose the experiment from the start screen (choose “PHYWE experiments”, search for “P2420264”, and click on the folders that contain this experiment). All necessary presetting will be loaded. You will need to modify settings during the experiment. To do so, click on the gear wheel button, choose “Sensors/Channels” and select “Xpert-Link” Channel CH1. For the first experiment (Charging curve of a capacitor), the measurement range has to be set to “10 mA”, and averaging of 15 values will reduce noise artefacts from your signal. Channel CH3 has to be set to a measurement range of 10 V, averaging 10 values. A sampling rate of 2 kHz will be sufficient.

To automatically start and stop the measurement, trigger settings need to be defined the following way:
- Start trigger CH1, threshold: 0.5 mA (rising)
- Pretrigger: 0.1 sec.
- Stop trigger CH1, threshold: 0.1 mA (falling)

When loading experiment presets, the first experiment can be performed without further modification. The correct settings for each measurement that follows will be given at the begin of each section.

During the experiment, you may wish to remove previous measurements from the diagram. To do so, click on the “x” symbol next to the measurement name in the diagram caption, or delete all of the measurements from the diagram by clicking on the “x” symbol in the analysis tools tab in the diagram. You can export your data by clicking on the “Data pool” symbol, and download your measurements from the data pool by clicking on the download symbol.

If you accidentally closed the diagram, you can either reload the predefined settings, or click on the function generator or on the virtual channel in the devices bar, and then click on “diagram”, or drag and drop the diagram onto the workspace. Further channels can be added by drag and drop into the diagram.

Start the measurement by clicking on the “Record” button. By closing the switch (click on Xpert-Link switch button), the measurement will be triggered. As the stop trigger is used, the measurement terminates automatically.

The curve you obtained may look like this:
Point the mouse cursor to your data in order to determine the half-value time of the recorded voltage curve. You can also use the survey tool (analysis toolbar) for this purpose.

Be aware that the origin of the time scale may not be necessarily identical with the origin of the desired signal (due to noise). If these differ make sure to subtract the offset from the half-life time.

Remove the capacitor from the circuit, and plug in a different capacitor at the same position. Repeat the measurement for all capacitors that are provided with this experiment and keep a record of the respective half-value time data. You will need these values when calculating the capacitance.

Use the short-circuit plug to discharge the capacitors in between measurements (short-circuit the cables for the voltage measurement that are connected parallely to the capacitor).

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2. Inductance measurement

Now plug the 900 turn coil into the circuit where the capacitor has been. Turn the On/Off switch off. Replace the 1 kΩ resistor with a 47 Ω resistor (see Fig. 1). This resistor is needed to limit the current to the selected measurement range.
You can clear the diagram or close and create a new diagram by clicking on the Xpert-Link symbol and the desired channel in the devices bar, and then clicking on “diagram” (or drag and drop the diagram onto the workspace). The second channel can be added by drag and drop into the diagram you just created.

For the inductance measurement set the parameters as follows:

CH1 I1 100 mA, Average: 15 values,
CH3 U1 10 V, Average: 1 value.
Sampling rate: 50 kHz.
Trigger settings: Same as for capacitance measurement.

Make sure to check that the sampling rate is at least 100 kHz, as the switching of the inductance on occurs a much smaller time scale.

Start the measurement by clicking on the “Record” button. Close the switch by clicking the switch button, re-open and stop the measurement. Use the fitting button (analysis tools) to match the measurement representation to the size of the diagram.

Your result may look like this:

Determine the half-life time of the switch-on process (use survey tool, as shown in Fig. Fehler: Referenz nicht gefunden). Use the current curve for evaluation. In the example, a half-life time of 0.38 ms was measured.
Try again, if bouncing of the switch appears and distorts the curves. Be aware that the origin of the time scale may not be necessarily identical with the origin of the desired signal (due to noise). If these differ make sure to subtract the offset from the half-life time.

3. Resistance measurement

The values “1 kΩ” and “47 Ω”, as printed on the resistor housing, can be used for evaluation as they are sufficiently precise. Nevertheless, you may wish to determine the exact values in order to minimize measurement errors when calculating the capacitance and inductance.

To do so, modify the setup according to Fig. 7.

![Resistance measurement circuit](image)

The program settings have to be modified for both of the resistors. Choose a measurement range of 10 mA (CH1) for the measurement of the 1 kΩ resistor, and a measurement range of 100 mA (CH1) for the measurement of the 47 Ω resistor.

The use of the "Average" option in both the current and voltage settings accounts for the suppression of noise and thus enhances the reliability of the measurement. The number of decimal places should be 2, as already provided in the experiment settings. You may wish to increase the number of decimal places to 2 in order to provide higher accuracy. To modify the Xpert-Link channel settings click on the Xpert-Link symbol in the devices bar, and enter the settings menu by clicking on the gear-wheel symbol. Confirm your entries by clicking “Apply” and/or “Ok”. The sampling rate can be set to a lower value, e.g. 2 kHz.

The current and voltage values that are visible in the digital display can now be used to calculate the exact resistance via Ohm’s law.

Your data may look like this:
Fig. 8: Resistance measurement (here: 1 kΩ resistor).

Repeat the measurement for the 47 Ω resistor. You can also use this method to determine the exact value of the resistance of the inductance, $R_L$. 
Theory and evaluation

1. Capacitance measurement

The voltage $U_c$ on capacitance $C$ with charge

$$Q(t) = \int_0^t I(t) \, dt = U_c(t) = \frac{Q(t)}{C}$$

The voltage on the resistance $R$ is with current $I(t) = \frac{dQ}{dt}$

$$U_R(t) = R \cdot I(t) = R \frac{dQ}{dt}$$

The constant voltage $U$ is switched on at $t = 0$ and

$$U = U_C(t) + U_R(t) = \frac{Q(t)}{C} + R \frac{dQ}{dt}$$

This differential equation with the boundary value $Q(0) = 0$ (the capacitor is discharged at the beginning) is solved by

$$Q(t) = U C \left(1 - e^{-\frac{1}{RC} t}\right)$$

And thus

$$I(t) = \frac{U}{R} e^{-\frac{1}{RC} t} \quad \text{and} \quad U_C(t) = U \left(1 - e^{-\frac{1}{RC} t}\right)$$

The halftime value is then given by

$$t_{1/2} = RC \cdot \ln(2)$$

The example measurement given in Fig 3 yields values of

$$R = \frac{5.03 \, \text{V}}{5.25 \, \text{mA}} = 958 \, \Omega$$

which is close to the imprinted value of 1 kΩ, and

$$C = \frac{t_{1/2}}{R \ln(2)} = \frac{0.038}{958 \cdot 0.693} = 45.2 \cdot 10^{-6} \, \text{F} = 45.2 \, \mu \text{F}$$

which is in accordance with the imprinted value of 47 μF and the usual tolerances of electrolytic capacitors (usually ±20%).

Repeat the evaluation for all the capacitors that are provided with this experiment, and discuss the results in your report.

2. Inductance measurement

The voltage $U_L$ of an inductance $L$ with the inner resistance $R_L$ is

$$U(t) = L \cdot \frac{dI}{dt} + R_L \cdot I(t)$$

and the voltage in the circuit with the shunt resistance $R_s$ is constant after switching on

$$U = U_{R_s}(t) + U_L(t) = (R_s + R_L) \cdot I(t) + L \cdot \frac{dI}{dt}$$

This differential equation with the boundary value $I(0) = 0$ has the solution

$$I(t) = \frac{U}{R_s} \left(1 - e^{-\frac{R_s}{L} t}\right)$$

$$\frac{dI}{dt} = \frac{U}{R_s} \frac{R}{L} \cdot e^{-\frac{R_s}{L} t}$$

Remark: We can measure in the experiment the voltage on the coil
\[ U_L = U \cdot e^{-\frac{R}{L} t} + \frac{U R_L}{R} \left(1 - e^{-\frac{R}{L} t}\right) \]

\[ = U \left(\frac{R_L}{R} + \left(1 - \frac{R_L}{R}\right) \cdot e^{-\frac{R}{L} t}\right) \]

But for evaluation the simpler formula for the current \( i(t) \) is used.

You can either use the values you determined in the third part of the experiment, or use the imprinted values on the hardware components, \( R_s = 47 \, \Omega \) and \( R_L = 6 \, \Omega \), which yields a total value of \( R = 53 \, \Omega \).

The half-time value is then

\[ t_{1/2} = \frac{R}{L} \cdot \ln(2) \]

And so

\[ L = \frac{t_{1/2} R}{\ln(2)} = \frac{0.38 \times 10^{-3} \times 53 \Omega}{0.6931} = 29.1 \, m\Omega \]

what corresponds well with the imprinted value of 24 mH on the coil within measurement errors.