High-pass and low-pass filters with Cobra4 Xpert-Link
(Item No.: P2440964)

Curricular Relevance

<table>
<thead>
<tr>
<th>Area of Expertise:</th>
<th>Education Level:</th>
<th>Topic:</th>
<th>Subtopic:</th>
<th>Experiment:</th>
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<tr>
<td>Physics</td>
<td>University</td>
<td>Electricity and Magnetism</td>
<td>Inductance, Electromagnetic Oscillations, AC Circuits</td>
<td>High-pass and low-pass filters with Cobra4 Xpert-Link</td>
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</table>

Difficulty | Preparation Time | Execution Time | Recommended Group Size
---|------------------|----------------|------------------|
Difficult | 10 Minutes | 20 Minutes | 2 Students

Additional Requirements:
- Software measureLAB
- PC with USB port, Windows 7 or higher

Experiment Variations:

Keywords:
- Circuit, Resistance, Capacitance, Inductance, Capacitor, Coil, Phase displacement, Filter, Kirchhoff's laws, Bode diagram

Overview

Short description

Principle
A coil, a capacitor, an ohmic resistance and combinations of these components are investigated for their filter characteristics as a function of frequency.

Fig. 1: Experimental set-up (here: two CR networks).
Equipment

<table>
<thead>
<tr>
<th>Position No.</th>
<th>Material</th>
<th>Order No.</th>
<th>Quantity</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Cobra4 Xpert-Link</td>
<td>12625-99</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Cobra4 Xpert-Link set of cables</td>
<td>12625-10</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>PHYWE Digital Function Generator, USB, incl. Cobra4 software</td>
<td>13654-99</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Connection box</td>
<td>06030-23</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>Coil, 300 turns</td>
<td>06513-01</td>
<td>1</td>
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<tr>
<td>6</td>
<td>Resistor 47 Ohm, 1 W, G1</td>
<td>39104-62</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>Resistor 1 kOhm, 1 W, G1</td>
<td>39104-19</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>Capacitor 1 µF / 100 V, G2</td>
<td>39113-01</td>
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<tr>
<td>9</td>
<td>Capacitor 2.2 µF / 100 V, G2</td>
<td>39113-02</td>
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<td>10</td>
<td>Short-circuit plug, white</td>
<td>06027-06</td>
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Additionally required:

- Software measureLAB
- PC with Windows 7 or higher

Tasks

Determine the ratio of output voltage to input voltage for

1. RC / CR networks
2. RL / LR networks
3. CL / LC networks
4. two CR networks connected in series

and determine the resonance frequency of the CL / LC network.
Set-up and procedure

Set up the equipment according to the circuit diagrams provided below. During this experiment, the set-up will be modified in various ways in order to explore the different types of network circuits. Please refer to the circuit diagrams provided below for the respective configurations. Start with the CR network configuration using the 1 μF capacitor and 47 Ω resistor. The different combinations of the provided capacitors and resistors can be used for variations of this experiment.

You can use the BNC output of the digital function generator to directly feed in the function generator signal into the Xpert-Link channel CH4 via BNC cable as this signal is identical to the signal of the 4 mm output sockets (as depicted in Fig. Fehler: Referenz nicht gefunden).

Now connect both the Xpert-Link and the function generator to your computer.

Start the software “measureLAB”, and choose the experiment from the start screen (choose “PHYWE experiments”, search for “P2440964”, and click on the folders that contain this experiment). All necessary presetting will be loaded. If you wish to define your own settings, click on the gear wheel button, choose “Sensors/Channels” and select “Xpert-Link” Channel CH3. For the first experiment (CR network), the measurement range has to be set to “10 V”, as this range corresponds to the function generator output range, and averaging of 50 values will reduce noise artefacts from your signal. In order to obtain effective values, “TRMS” needs to be selected. The same settings need to be applied to CH4. The function generator settings can be modified when choosing “Sensors/Channels” and “Function generator”. For the first experiment, a frequency ramp in the range 0 Hz - 10 kHz is suitable (increment: 1 Hz, Pause time: 1 ms). A sampling rate of 1 kHz will be sufficient. When loading experiment presettings, all the experiments can be performed without further modification, except for the LC/-CL-network (see below).

Settings overview:

CH3: 10 V, Average 50 values, TRMS active
CH4: 10 V, Average 50 values, TRMS active

Function generator (Power output): Frequency ramp 0 Hz-10 kHz, Increment 1 Hz, Pause time 1 ms, Amplitude pp. 5 V, Offset 0 V.
Sampling rate:1 kHz.

For a live measurement of the output-to-input voltage ratio virtual channels are used. When loading the presettings, a predefined virtual channel “Ratio” will measure the ratio CH3/CH4 as a function of the frequency.

If you wish to define own virtual channels, click on the button in the devices bar, then click on the “+” button, and the virtual channel settings menu will open. This menu allows for the definition of new virtual channels by giving values for number of decimal places, unit, channel name and symbol, and by giving the formula that defines the way the channel will be calculated. Click on the channel symbols given below in order to enter these channels into the formula, or drag and drop them into the formula input mask. You can use arithmetic operations and trigonometric functions for the definition of virtual channels by entering the desired operations with your keyboard. If an error in the formula occurs, e.g. brackets are not closed, a red frame will indicate that you need to correct the entry before performing the measurement.

In all the experiments described below, the virtual channel is defined as CH3/CH4, thus by using the provided presettings, no virtual channel modification is necessary.

Start the measurement by clicking the “Record” button. Your result may look like depicted in Fig. 2.

During the experiment, you may wish to remove previous measurements from the diagram. To do so, click on the “x” symbol next to the measurement name in the diagram caption, or delete all of the measurements from the diagram by clicking on the “x” symbol in the analysis tools tab in the diagram. You can export your data by clicking on the “Data pool” symbol, and download your measurements from the data pool by clicking on the download symbol. Now perform measurements for the different types of networks, as described below.

If you accidentally closed the diagram, you can either reload the predefined settings, or click on the function generator or on the virtual channel in the devices bar, and then click on “diagram”, or drag and drop the diagram onto the workspace. The second channel can be added by drag and drop into the diagram, and the frequency can be selected as the channel that defines the x-axis by choosing “frequency” from the channel list in the drop down menu of the diagram representation tab in the diagram.

1. RC / CR network:

Record curves for the following networks with the 47 Ω and 1 kΩ resistors and the 1 μF and 2.2 μF capacitors.
Your results may look like this:

Fig. 2: Example data: CR network.

Your results may look like this:
2. **RL / LR network:**

Record curves for the following networks with the 300 turn coil and the 47 Ω resistor:

Your results may look like this:

![RL network diagram](image)

![RL network graph](image)

**Fig. 3:** Example data: RC network.

**Fig. 4:** Example data: RL network.
3. LC / CL network:

For this measurement, the measuring range of the CH3 U1 has to be set to 30 V. To do so, click on the Xpert-Link symbol in the devices bar, then click on the gear-wheel button. Select “30 V” for channel CH3. This is necessary as the signal will exceed 10 V by far.

Record curves for the following networks with the 300 turn coil and the 1 μF and 2.2 μF capacitors.
Your results may look like this:

![Diagram](image)

Fig. 6: LC and CL network (here: Capacitance 1 µF).

Note the value for the resonance frequency from these measurements. You can zoom into the diagram by using the analysis tools (magnifying glass or cropping tool) or by scrolling with your mouse scroll wheel while the mouse cursor tool is selected. To return to a representation of the complete data set click on the fitting tool next to the mouse cursor tool.

4. Two CR networks connected in series:

For this measurement, set the measurement range for CH3 U1 back to 10 V. To do so, click on the Xpert-Link symbol in the devices bar, then click on the gear-wheel button. Select “10 V” for channel CH3.

Record curves for this network, e.g. $C_1 = 1 \, \mu \text{F}$, $R_1 = 47 \, \Omega$, $C_2 = 2.2 \, \mu \text{F}$, $R_2 = 1 \, \text{k} \Omega$

![Two CR networks in series](image)

Fig. 7: Setup for two CR networks on the connection box. Black cables are connected to the function generator output, blue cables to CH 3. The red cable is used to connect the resistors.
Plot the ratio output to input voltage for the recorded curves. The result may look like this:

Fig. 8: Example data: Two RC networks.
Theory and evaluation

The voltage $U_C$ on a capacitance $C$ with charge

$$Q(t) = \int_0^t I(t) \, dt$$

is

$$U_C(t) = \frac{Q(t)}{C}.$$ 

The voltage on the resistance $R$ is with current $I(t) = \frac{dQ}{dt}$

$$U_R(t) = R \cdot I(t) = R \frac{dQ}{dt}.$$ 

$$U(t) = U_C(t) + U_R(t) = \frac{Q(t)}{C} + R \frac{dQ}{dt} = U_0 \cos(\omega t).$$

for a resistor and a capacitance in series connected to an ac voltage source. Differentiating this equation yields

$$\frac{I}{C} + R \frac{dI}{dt} = -\omega \cdot U_0 \sin(\omega \cdot t).$$

This differential equation has the solution

$$I(t) = I_0 \cos(\omega \cdot t + \varphi)$$

with

$$\tan(\varphi) = \frac{1}{\omega \cdot CR} > 0$$

i.e. the current is ahead of the voltage and

$$I_0 = \frac{U_0}{\sqrt{R^2 + \left(\frac{1}{\omega \cdot C}\right)^2}}$$

The voltage on the resistor is $U_R = R \cdot I(t)$ and for the amplitudes is the ratio output to input in the case of a CR filter then

$$\frac{R \cdot I_0}{U_0} = \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} = \frac{1}{\sqrt{1 + \frac{1}{\omega^2 C^2 R^2}}}$$

which is low for low frequencies and reaches one for high frequencies, i.e. the CR filter is a high pass filter.

The voltage amplitude on the capacitor is with the impedance of the capacitor
1. **RC Filter**:

The ratio output to input is then

\[
\frac{U_{C_0}}{U_0} = \frac{1}{\frac{1}{\omega \cdot C} \sqrt{R^2 + \frac{1}{\omega^2 \cdot C^2}}}
\]

so the ratio output to input is then

\[
\frac{U_{C_0}}{U_0} = \frac{1}{\sqrt{\omega^2 C^2 R^2 + 1}}
\]

which starts for low frequencies at one and drops to zero for high frequencies like \(\frac{1}{\omega}\), i.e. the RC filter is a low pass filter.

\[
\hat{R}_C = \frac{1}{\omega \cdot C} = \frac{U_{C_0}}{I_0}
\]

2. **RL / LR Filter**:

Inductance \(L\) and resistance \(R\) are connected in series to an ac source

\[
U(t) = U_0 \cos(\omega \cdot t) = R \cdot I(t) + \frac{L}{\omega} \frac{dI}{dt}
\]

This differential equation has the solution

\[
I(t) = I_0 \cos(\omega \cdot t + \varphi)
\]

with

\[
I_0 = \frac{U_0}{\sqrt{R^2 + \omega^2 L^2}}
\]

and

\[
\tan(\varphi) = \frac{-\omega \cdot L}{R} < 0.
\]

i.e. the voltage is ahead of the current.

The voltage on the resistor is \(U_R = R \cdot I(t)\)

and for the amplitudes is the ratio output to input in the case of a LR filter then

\[
\frac{R \cdot I_0}{U_0} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \frac{1}{\sqrt{1 + \frac{\omega^2 L^2}{R^2}}}
\]

which is one for low frequencies and reaches zero for high frequencies, i.e. the LR filter is a low pass filter.

The voltage amplitude on the inductance is with the impedance of the coil

\[
\hat{R}_L = \omega \cdot L = \frac{U_{I_0}}{I_0}
\]
\[ U_{L_0} = \frac{U_0 \cdot \omega \cdot L}{\sqrt{R^2 + \omega^2 \cdot L^2}} \]

so the ratio output to input is then

\[ \frac{U_{L_0}}{U_0} = \frac{1}{\sqrt{\frac{R^2}{\omega^2 \cdot L^2} + 1}} \]

which starts for low frequencies at zero and rises to one for high frequencies, i.e. the RL filter is a high pass filter.

3. LC / CL filter

Inductance \( L \) and capacitance \( C \) are connected in series to an ac source

\[ U(t) = U_0 \cos(\omega \cdot t) = \frac{Q(t)}{C} + L \frac{dI}{dt} \]

This differential equation has the solution

\[ I(t) = I_0 \cos(\omega \cdot t + \varphi) \]

with

\[ I_0 = \frac{U_0}{\omega \cdot L} - \frac{1}{\omega \cdot C} \]

and

\[ \tan(\varphi) = \frac{1}{\omega \cdot C} - \omega \cdot L. \]

i.e. the current is ahead of the voltage for low frequencies and behind for high frequencies and there is no phase shift in the case of resonance.

The voltage amplitude on the capacitor is with the impedance of the capacitor

\[ \hat{R}_C = \frac{1}{\omega \cdot C} = \frac{U_{C_0}}{I_0} \]

\[ U_{C_0} = \left| \frac{U_0}{\omega \cdot C} \cdot \frac{1}{\omega \cdot L - \frac{1}{\omega \cdot C}} \right| = \left| \frac{U_0}{\omega^2 LC - 1} \right| \]

so the ratio output to input is then

\[ \frac{U_{C_0}}{U_0} = \left| \frac{1}{\omega^2 LC - 1} \right| \]

which starts for low frequencies at one and has a pole at \( \omega = \frac{1}{\sqrt{LC}} \) (Thomson equation) and goes to zero for high frequencies.
like $\frac{1}{\omega^2}$ i.e. the LC filter is a low pass filter.

The voltage amplitude on the inductance is with the impedance of the coil

$$\tilde{R}_L = \omega \cdot L = \frac{U_{L_0}}{I_0}$$

$$U_{L_0} = \frac{U_0 \cdot \omega \cdot L}{\omega \cdot L - \frac{1}{\omega \cdot C}}$$

so the ratio output to input is then

$$\frac{U_{L_0}}{U_0} = \left| \frac{1}{1 - \frac{1}{\omega^2 LC}} \right|$$

which starts for low frequencies at zero and has a pole at $\omega = \frac{1}{\sqrt{LC}}$ (Thomson equation) and does not drop under one for high frequencies, i.e. the CL filter is a high pass filter.

For the resonance frequency of the LC/CL-network in the setup using $L = 2 \text{ mH}$ and $C = 1 \mu\text{F}$, the Thomson equation predicts a resonance frequency of $f = \frac{\omega}{2\pi} = 3559 \text{ Hz}$. The data in Fig. 6 yields a resonance frequency of $3408 \text{ Hz}$, which is in good agreement with the predicted value.
Report: High-pass and low-pass filters with Cobra4 Xpert-Link