Atomic spectra of two-electron systems: He, Hg
(Item No.: P2510800)

Curricular Relevance

<table>
<thead>
<tr>
<th>Area of Expertise:</th>
<th>Education Level:</th>
<th>Topic:</th>
<th>Subtopic:</th>
<th>Experiment:</th>
</tr>
</thead>
<tbody>
<tr>
<td>ILIAS</td>
<td>Physik</td>
<td>Hochschule</td>
<td>Moderne Physik</td>
<td>Quantenphysik</td>
</tr>
</tbody>
</table>

**Difficulty**  
Difficult

**Preparation Time**  
1 Hour

**Execution Time**  
2 Hours

**Recommended Group Size**  
2 Students

**Additional Requirements:**

**Experiment Variations:**

**Keywords:**
Parahelium, orthohelium, exchange energy, spin, angular momentum, spinorbit interaction, singlet series, triplet series, multiplicity, Rydberg series, selection rules, forbidden transitions, metastable state, energy level, excitation energy

Overview

**Short description**

**Principle**
The spectral lines of He and Hg are examined by means of a diffraction grating. The wavelengths of the lines are determined from the geometrical arrangement and the diffraction grating constants.

Fig.1: Experimental set up for measuring the spectra of He and Hg.
Equipment

<table>
<thead>
<tr>
<th>Position No.</th>
<th>Material</th>
<th>Order No.</th>
<th>Quantity</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Spectrum tube, mercury</td>
<td>06664.00</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Spectrum tube, helium</td>
<td>06668.00</td>
<td>1</td>
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<tr>
<td>3</td>
<td>Holders for spectral tubes, 1 pair</td>
<td>06674.00</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Cover tube for spectral tubes</td>
<td>06675.00</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>Connecting cord, 30 kV, l = 1000 mm</td>
<td>07367.00</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>Object holder, 55 cm</td>
<td>08041.00</td>
<td>1</td>
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<tr>
<td>7</td>
<td>Diffraction grating, 600 lines/mm</td>
<td>08546.00</td>
<td>1</td>
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<tr>
<td>8</td>
<td>High voltage supply unit, 0-10 kV</td>
<td>13670.93</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>Insulating support</td>
<td>06020.00</td>
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<tr>
<td>10</td>
<td>Tripod base -PASS-</td>
<td>02002.55</td>
<td>1</td>
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<td>11</td>
<td>Barrel base -PASS-</td>
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<td>12</td>
<td>Support rod -PASS-, square, l = 400 mm</td>
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<td>13</td>
<td>Right angle clamp -PASS-</td>
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<td>3</td>
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<td>14</td>
<td>Stand tube</td>
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<tr>
<td>15</td>
<td>Meter scale, demo, l = 1000 mm</td>
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<td>1</td>
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<tr>
<td>16</td>
<td>Cursors, 1 pair</td>
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<td>1</td>
</tr>
<tr>
<td>17</td>
<td>Measuring tape, l = 2 m</td>
<td>09936.00</td>
<td>1</td>
</tr>
</tbody>
</table>

tasks

1. Determination of the wavelengths of the most intense spectral lines of $\text{He}$.  
2. Determination of the wavelengths of the most intense spectral lines of $\text{Hg}$.  

Set-up and procedure

The experimental set-up is shown in Fig. 1. Helium or mercury spectral tubes connected to the high voltage power supply unit are used as a source of radiation. The power supply is adjusted to about 5 kV. The scale is attached directly behind the spectral tube in order to minimize parallax errors. The diffraction grating should be set up at about 50 cm and at the same height as the spectral tube. The grating must be aligned so as to be parallel to the scale.

The luminous capillary tube is observed through the grating. The room is darkened to the point where it is still possible to read the scale. The distance $2l$ between spectral lines of the same color in the right and left first order spectra are read without moving one’s head. The distance $d$ between the scale and the grating is also measured.

The individual lines (first order) of the spectral lamp are observed by means of the grating and the distance $2l$ between equal lines is determined with the metre scale.
**Theory and evaluation**

1. If light of wavelength $\lambda$ falls on a grating having a grating constant $k$, it is diffracted. Intensity maxima occur if the angle of diffraction which satisfies the condition:

$$n \cdot \lambda = k \cdot \sin \varphi; \ n = 0, 1, 2 \ldots$$

From Fig. 2, we have:

$$\sin \varphi = \frac{l}{\sqrt{d^2 + l^2}}$$

and hence

$$\lambda = \frac{k l}{\sqrt{d^2 + l^2}}$$

for the first-order diffraction.

![Fig. 2: Diffraction of light of wavelength $l$ at the grating.](image)

2. Excitation of the He and Hg atoms results from electron impact. The energy difference produced when electrons revert from the excited state $E_1$ to the ground state $E_0$ is emitted as a photon with a frequency $f$.

$$hf = E_1 - E_0$$

where

$$h = \text{Planck's constant}$$

$$= 6.63 \cdot 10^{-34} \text{ Js.}$$

The Hamiltonian operator (non-relativistic) for the two electrons and of the He atom is:

$$H = -\frac{\hbar^2}{2m} \Delta_1 - \frac{\hbar^2}{2m} \Delta_2 - \frac{2e^2}{|r_1|} - \frac{2e^2}{|r_2|} + \frac{2e^2}{|r_1 - r_2|}$$

where $\hbar = \frac{\hbar}{2\pi}$.

$m$ and $e$ represent the mass and charge of the electron respectively,

$$\Delta_i = \frac{d^2}{dx_i^2} + \frac{d^2}{dy_i^2} + \frac{d^2}{dz_i^2}$$

is the Laplace operator, and $r_{i\rightarrow}$ is the position of the $i$-th electron. The spin-orbit interaction energy

$$E_{so} \propto \frac{e^2}{4 \cdot (137)}$$

was ignored in the case of the nuclear charge $Z = 2$ of helium, because it is small when $Z$ is small.

If we consider

$$\frac{e^2}{|r_1 - r_2|}$$

as the electron-electron interaction term, then the eigenvalues of the Hamiltonian operator without interaction are those of the hydrogen atom:

$$E_{i,m}^0 = -\frac{me^4}{8\hbar^2} \left( \frac{1}{n_i} + \frac{1}{m_i^2} \right)$$
As the transition probability for simultaneous two-electron excitation is very much less than that for one-electron excitation, the energy spectrum of the system without interaction is:

\[ E_{i,m}^0 = -\frac{me^4}{8\hbar^2} \left( 1 + \frac{1}{m^2} \right) \]

\[ m = 1, 2. \]

The interaction term removes out the angular momentum degeneracy of the pure hydrogen spectrum and the exchange energy degeneracy. There results an energy adjustment:

\[ E_{n\ell \sigma}^1 = \langle \phi_{n\ell \alpha}^\pm | \frac{e^2}{r_{\alpha \beta}} | \phi_{n\ell \alpha}^\mp \rangle = C_{n\ell} \pm A_{n\ell} \]

in which \( \phi_{n\ell \alpha}^\pm \) are the antisymmetrized undisturbed 2-particle wave functions with symmetrical (\( \phi^+ \)) or antisymmetrical (\( \phi^- \)) position component, \( \ell^* \) is the angular momentum quantum number, and \( \alpha \) is the set of the other quantum numbers required.

In the present case, the orbital angular momentum of the single electron \( \ell \) is equal to the total angular momentum of the two electrons \( L \), since only one-particle excitations are being considered and the second electron remains in the ground state (\( \ell = 0 \)).

\( C_{n\ell} \) and \( A_{n\ell} \) are the Coulomb and exchange energy respectively. They are positive. Coupling the orbital angular momentum \( L \) with the total spin \( S \) produces for \( S = 0 \), i.e. \( \phi^+ \), a singlet series and for \( S = 1 \), i.e. \( \phi^- \), a triplet series. Because of the lack of spin-orbit interaction, splitting within a triplet is slight. As the disturbed wave functions are eigenfunctions for \( S^2 \) and as \( S^2 \) interchanges with the dipole operator, the selection rule

\[ \Delta S = 0 \]

(which is characteristic for 2-electron systems with a low nuclear-charge number) results and forbids transitions between the triplet and singlet levels.

In addition, independent of the spin-orbit interaction, the selection rule for the total angular momentum

\[ \Delta J = 0, \pm 1 \]

applies except where

\[ J = 0 \rightarrow J' = 0. \]

If the spin-orbit interaction is slight, then

\[ \Delta L = 0, \pm 1 \]

applies.

Detailed calculations produce the helium spectrum of Fig. 3.
The following table gives the measured lines:

<table>
<thead>
<tr>
<th>Colour</th>
<th>λ/nm</th>
<th>Transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td>665 ± 2</td>
<td>$^3P_1 \rightarrow ^1D_3$</td>
</tr>
<tr>
<td>yellow-orange</td>
<td>586 ± 2</td>
<td>$^3D_2 \rightarrow ^1P_1$</td>
</tr>
<tr>
<td>green</td>
<td>501 ± 2</td>
<td>$^3P_1 \rightarrow ^1D_3$</td>
</tr>
<tr>
<td>blue-green</td>
<td>490 ± 2</td>
<td>$^4D_2 \rightarrow ^1P_1$</td>
</tr>
<tr>
<td>blue</td>
<td>470 ± 3</td>
<td>$^4S_2 \rightarrow ^3P_1$</td>
</tr>
<tr>
<td>violet</td>
<td>445 ± 1</td>
<td>$^4D_2 \rightarrow ^3P_1$</td>
</tr>
</tbody>
</table>

Table 1: Measured spectral lines of He and the corresponding energy-level transitions.

The magnitudes of the exchange interaction and the Coulomb interaction of the two electrons can be estimated by comparing the energies of the transitions:

$$3^1D_2 \rightarrow 2^1P_1$$
$$3^3D_2 \rightarrow 2^3P_1$$

or

$$4^1D_2 \rightarrow 2^1P_1$$
$$4^3D_2 \rightarrow 2^3P_1$$

3. Hg, likewise, is a 2-electron system and possesses the structure of 2 series.

The spin-orbit interaction, however, is relatively pronounced so that only the total angular momentum

$$J = L + S$$

is a “good” conservation parameter. Splitting inside the triplet is pronounced.

Moreover, the selection principle

$$\Delta S = 0$$

no longer applies since $S$ is no longer a “good” conservation parameter (transition from $L-S$ for the $j-j$ coupling).
Fig. 4: Spectrum of mercury.

The table below gives the lines obtained by experiment:

<table>
<thead>
<tr>
<th>Colour</th>
<th>( \lambda ) (\text{nm} )</th>
<th>Transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>yellow</td>
<td>581 ± 1</td>
<td>( 6^{1}D_{1} \rightarrow 6^{1}P_{1} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( 6^{1}D_{1} \rightarrow 6^{1}P_{1} )</td>
</tr>
<tr>
<td>green</td>
<td>550 ± 1</td>
<td>( 7 \ 3S_{1} \rightarrow 6^{1}P_{1} )</td>
</tr>
<tr>
<td>green</td>
<td>404 ± 2</td>
<td>( 8 \ 3S_{1} \rightarrow 6^{1}P_{1} )</td>
</tr>
<tr>
<td>blue</td>
<td>437 ± 2</td>
<td>( 7 \ 3S \rightarrow 6^{1}P_{1} )</td>
</tr>
</tbody>
</table>

Table 2: Measured spectral lines of \( \text{Hg} \) and the corresponding energy-level transitions.

Literature:

G. Herzberg, Atomic Spectra and Atomic Structure (Dover Publ.).

D.R. Bates, Quantum Theory II (Academic Press Inc.).